Manikanta. Surokasula

UNIT-V - THIN CYLINDERS & THICK CYLINDERS

In order to meet several requirements, the fluids are stored under pressure in pressure vessels (or) shells. Vessels of spherical and cylindrical form are used for storing fluids under pressure.

eg - steam boilers, air compressors, tanks & water tanks. Spheres are used for storing gas under pressure. If fluid is a gas, the pressure is constant in all parts of vessel. In case of liquid, the pressure is lowest at the top & increases with depth. When the vessels are empty, they are subjected to an atmospheric pressure both internally and externally and hence the resultant effect of atmospheric pressure is nil.

Thin cylindrical shells :-

A cylindrical vessel may be thin (or) thick depending upon the thickness of the plate in relation to the internal diameter of the cylinder. In thin cylinders, the stress may be assumed uniformly distributed over the wall thickness.

If the thickness of the wall of the cylindrical vessel is less than $\frac{1}{20}$ of its internal diameter the cylindrical vessel is known as a thin cylinder. Means, for thin cylinders, $t/d < \frac{1}{20}$.

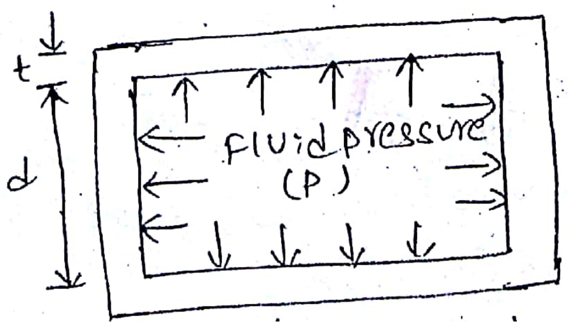
If the ratio of t/d is more than $\frac{1}{20}$, then cylindrical shell is known as thick cylinders.

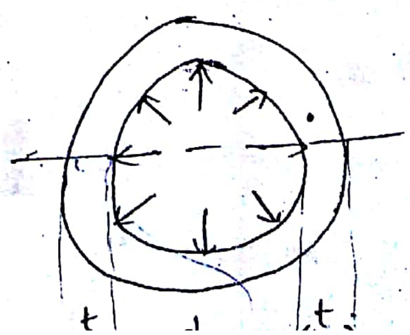
When thin cylinders are subjected to internal fluid pressures, the following types of stresses are developed.

1. Hoop (or) circumferential stresses : These act in a tangential direction to the circumference of the shell.
2. Longitudinal stresses : These act parallel to the longitudinal axis of the shell.
3. Radial stresses : These act radially and are too small and can be neglected.

These three stresses are mutually perpendicular and are principal stresses.

Thin cylindrical vessel subjected to internal pressure





Let d - internal diameter of the thin cylinder

t - thickness of the cylindrical wall.

p - Internal pressure of the fluid.

L - Length of the cylinder.

The forces due to pressure of the fluid acting vertically upwards & downwards on the thin cylinder, tend to burst the cylinder as shown in fig(a)

The forces, due to pressure of the fluid, acting at the ends of the thin cylinder, tend to burst the thin cylinder as shown in fig(b).

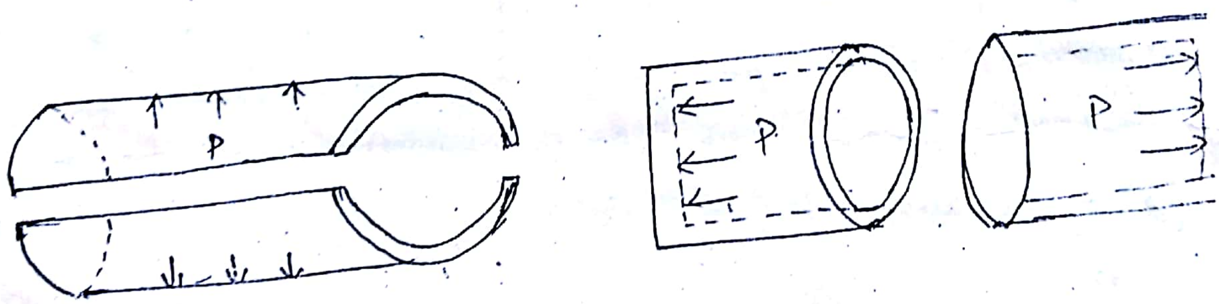
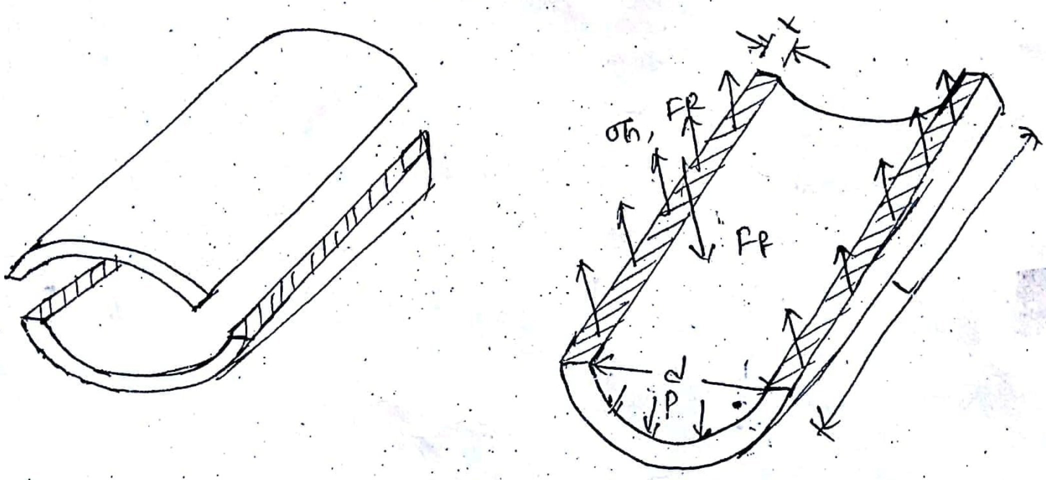


Fig. (a)

→ The circumferential stress developed due to the longitudinal joint and which is acting along the circumference of the cylinder.

→ The longitudinal stress developed due to the circumferential joint and which is acting along the length of the cylinder.

Expression for circumferential (or) hoop stress :



consider a thin cylindrical vessel subjected to an internal fluid pressure. The circumferential stress will be set up in the material of the cylinder, if the bursting of the cylinder takes place.

Let,

P = internal pressure of the fluid

d - internal diameter of the cylinder.

t - thickness of the cylindrical wall.

σ_h - circumferential (or) hoop stress

The bursting will takes place if the force due to fluid pressure is more than the resisting force due to circumferential stress set up in the material.

In the limiting case,

Fluid force = Resisting force

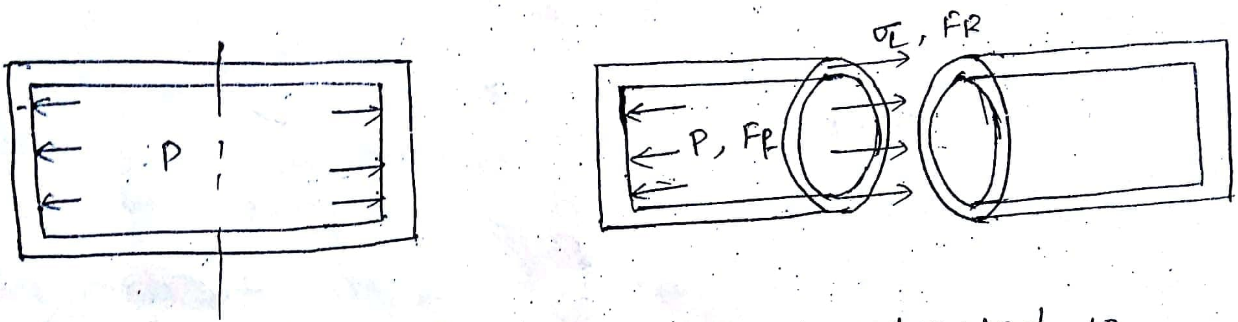
$F_P = F_R + F_R$

P x Area on which P is acting = 2 x σ_h x Area on which σ_h is acting

$$P \times (d \times \cancel{l}) = 2\sigma_h \times (\cancel{l} \times t)$$

$$\boxed{\sigma_h = \frac{Pd}{2t}}$$

Expression for longitudinal stress



Consider a thin cylindrical vessel subjected to internal fluid pressure. The longitudinal stress will be set up in the material of the cylinder, if the bursting of the cylinder takes place along the section perpendicular to the longitudinal axis.

Let p - internal pressure of the fluid
d - internal diameter of the cylinder
t - thickness of the cylindrical wall
σ_L - longitudinal stress.

The bursting will takes place, if the fluid force ~~is~~ due to the pressure acting on the ends of the cylinder is more than the resisting force due to longitudinal stress.

In limiting case,

Fluid force = Resisting force

P x Area on which P is acting = σ_L x Area on which σ_L is acting.

...l shell

$$P \times \frac{\pi}{4} d^2 = \sigma_L \times \pi d \cdot t$$

$$\boxed{\sigma_L = \frac{Pd}{4t}}$$

$\therefore \sigma_h > \sigma_L$ $\quad\quad$ $\sigma_h - \sigma_{max}$

$\sigma_h = 2\sigma_L$ $\quad\quad$ $\sigma_L - \sigma_{min}$

Hence in the material of the cylinder the permissible stress should be less than the circumferentia stress.

Maximum shear stress :

At any point in the material of the cylindrical shell, there are two principal stresses namely
1. circumferential stress
2. longitudinal stress.

These two stresses are tensile and perpendicular to each other.

$$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2}$$

$$= \frac{\sigma_h - \sigma_L}{2} = \frac{\frac{Pd}{4t} - \frac{Pd}{2t}}{2}$$

$$\boxed{\tau_{max} = \frac{Pd}{8t}}$$

Change in dimensions of a thin cylindrical shell due to an internal pressure :

A cylindrical shell, due to circumferential and longitudinal stresses, will increase in length as well as undergo a change in dimensions resulting in change of its volume.

Let,

p - internal fluid pressure

L - Length of cylindrical shell

d - Diameter of the cylindrical shell.

t - thickness of the cylindrical shell

σ_h - Hoop stress

σ_L - Longitudinal stress.

E - Young's modulus

μ - Poisson's ratio.

δd, δL & δV - change in diameter, length & volume of the cylinder respectively.

ε_h - circumferential ~~(or)~~ hoop strain

ε_L - Longitudinal strain.

$$\varepsilon_h = \frac{\sigma_h}{E} - \mu \times \frac{\sigma_L}{E} = \frac{\delta d}{d}$$

$$= \frac{pd}{2tE} - \mu \times \frac{pd}{4tE}$$

$$= \frac{pd}{2tE}\left[1 - \frac{\mu}{2}\right]$$

$$\varepsilon_L = \frac{\sigma_L}{E} - \mu \times \frac{\sigma_h}{E} \quad \cancel{\frac{\delta V}{L}}$$

$$\varepsilon_L = \frac{\delta L}{L}$$

$$\varepsilon_L = \frac{Pd}{4tE} - \mu \times \frac{Pd}{2tE}$$

$$= \frac{Pd}{2tE}\left[\frac{1}{2} - \mu\right]$$

Let

$d + \delta d$

$$\therefore \frac{\delta d}{d} = \frac{Pd}{2tE}\left(1 - \frac{\mu}{2}\right)$$

$$\boxed{\delta d = \frac{Pd^2}{2tE}\left[1 - \frac{\mu}{2}\right]}$$

$$\therefore \frac{\delta L}{L} = \frac{Pd}{2tE}\left(\frac{1}{2} - \mu\right)$$

$$\boxed{\delta L = \frac{PdL}{2tE}\left[\frac{1}{2} - \mu\right]}$$

Let

$d + \delta d$ - Final diameter of the cylinder

$L + \delta L$ - Final length of the cylinder

V - volume of the cylinder.

$$V = \frac{\pi}{4} d^2 L$$

Final volume $= \frac{\pi}{4}(d + \delta d)^2 (L + \delta L)$

$$= \frac{\pi}{4}(d^2 + \delta d^2 + 2d\delta d)(L + \delta L)$$

$$= \frac{\pi}{4}\left[d^2L + d^2\delta L + \cancel{L\delta d^2}^{\,0} + \cancel{\delta d^2 \delta L}^{\,0} + 2dL\delta d + \cancel{2d\,\delta d\,\delta L}^{\,0}\right]$$

(5)

∴ Smaller terms are neglected.

∴ Final volume $= \frac{\pi}{4}(d^2L + d^2\delta L + 2dL\delta d)$

Volumetric strain $= \dfrac{\text{change in volume}}{\text{original volume}}$

$$= \frac{\text{Final volume} - \text{original volume}}{\text{original volume}}$$

$$\varepsilon_v = \frac{\frac{\pi}{4}(d^2L + d^2\delta L + 2dL\delta d) - \frac{\pi}{4}d^2L}{\frac{\pi}{4}d^2L}$$

$$\varepsilon_v = \frac{\cancel{d^2L} + d^2\delta L + 2dL\delta d - \cancel{d^2L}}{d^2L}$$

$$= \frac{\cancel{d^2}\delta L}{\cancel{d^2}L} + \frac{2\cancel{dL}\delta d}{d^{\cancel{2}}\cancel{L}}$$

$$= \frac{\delta L}{L} + 2\frac{\delta d}{d}$$

$$\varepsilon_v = \varepsilon_L + 2\varepsilon_h$$

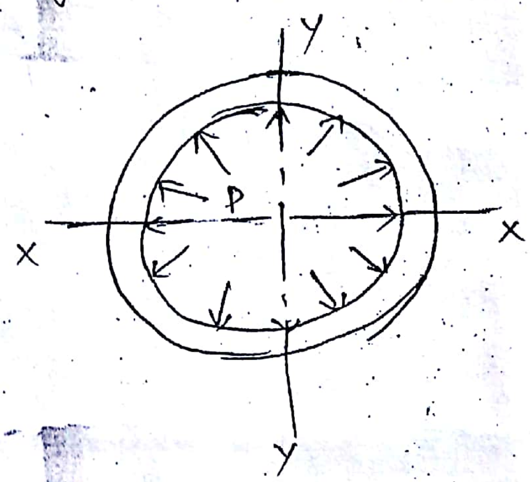
$$= \frac{Pd}{2tE}\left(\frac{1}{2} - \mu\right) + 2\frac{Pd}{2tE}\left(1 - \frac{\mu}{2}\right)$$

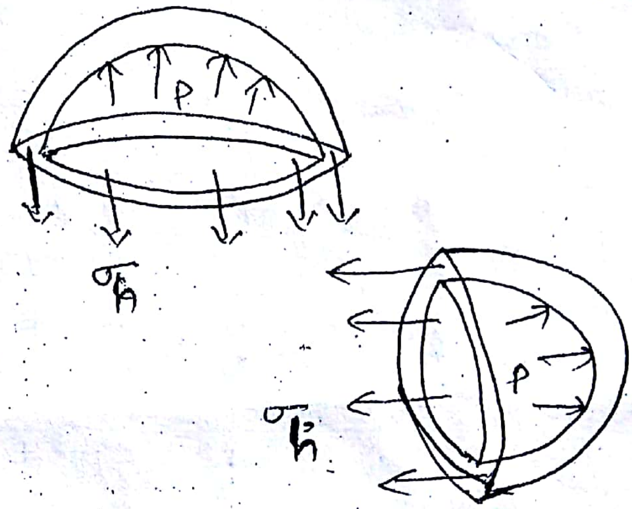
$$= \frac{Pd}{2tE}\left[\frac{1}{2} - \mu + 2 - \mu\right]$$

$$\boxed{\varepsilon_v = \frac{Pd}{2tE}\left[\frac{5}{2} - 2\mu\right]} = \frac{\delta V}{V}$$

Thin spherical shells

A thin spherical shell of internal dia. 'd' and thickness 't' is subjected to an internal fluid pressure 'p'. The fluid inside the shell has a tendency to split the shell into two hemispheres along X-X & Y-Y





Along X-X axis

In limiting case

fluid force = Resisting force

$$p \times \frac{\pi}{4} d^2 = (\sigma_h)_x \times \pi d t$$

$$\boxed{(\sigma_h)_x = \frac{Pd}{4t}}$$

Along Y-Y axis

In limiting case

fluid force = Resisting force

$$p \times \frac{\pi}{4} d^2 = (\sigma_h)_y \times \pi d t$$

$\therefore (\sigma_h)_x$ & $(\sigma_h)_y$ will be perpendicular to each other and are equal.

<u>Change in dimensions of a thin spherical shell due to an internal pressure</u> :

strain in any direction

$$\epsilon_h = \frac{\delta d}{d}$$

$$\epsilon_h = \frac{(\sigma_h)_x}{E} - \mu \times \frac{(\sigma_h)_y}{E}$$

$$= \frac{pd}{4tE} - \mu \times \frac{pd}{4tE}$$

$$\epsilon_h = \frac{pd}{4tE}(1-\mu)$$

$$\therefore \frac{\delta d}{d} = \frac{pd}{4tE}(1-\mu)$$

$(d+\delta d)^3$

$$\boxed{\delta d = \frac{pd^2}{4tE}(1-\mu)}$$

$d+\delta d$,

$L+\delta l$

volumetric strain $\epsilon_v = \frac{\delta V}{V}$

volume of the sphere $(V) = \frac{\pi}{6} d^3$

$$\epsilon_v = \frac{\delta\left(\frac{\pi}{6} d^3\right)}{V}$$

$$= \frac{\cancel{\frac{\pi}{6}} \times 3\cancel{d^2} \times \delta d}{\cancel{\frac{\pi}{6}} \cancel{d^3}}$$

$$= 3 \times \frac{\delta d}{d}$$

$\varepsilon_v = 3 \times \varepsilon_h$

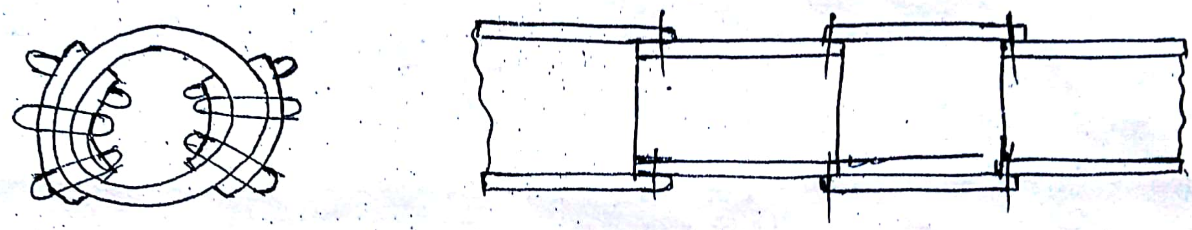
$$\boxed{\frac{\delta V}{V} = \varepsilon_v = 3 \times \frac{pd}{4tE}(1-\mu)}$$

the

no

Riveted cylindrical boilers :

A boiler of the desired capacity can be made by bending plates to the required diameter and connecting them, usually by a butt joint.



(b3+b) The desired length of the boiler can be obtained by connecting individually fabricated shells by usually a lap joint.

b3+b In the case of riveted shells the circumferential

18+1 and longitudinal stresses are greater. This is due to weakening of the plates due to rivet holes.

If $\eta_{l.J}$ is the efficiency of the longitudinal joints,

$$\boxed{\sigma_h = \frac{pd}{2t \times \eta_{l.J}}}$$

If $\eta_{c.J}$ is the efficiency of the circumferential joints,

$$\sigma_l = \frac{pd}{4t \times \eta_{c.J}}$$

The thickness of the shell in order the hoop stress may not exceed the permissible stress is.

$$t = \frac{pd}{2\sigma \times \eta_{L.J}}$$

Thick cylinders

Thick cylinders are the cylindrical vessels, containing fluid under pressure and whose t/d ratio is not small $(t/d \geq \frac{1}{20})$.

→ Radial stress is not neglisible, it varies from the inner surface where it is equal to the magnitude of the fluid pressure to the outer surface where usually it is equal to zero if exposed to the atmosphere.

→ circumferential stress also varies along the thickness.

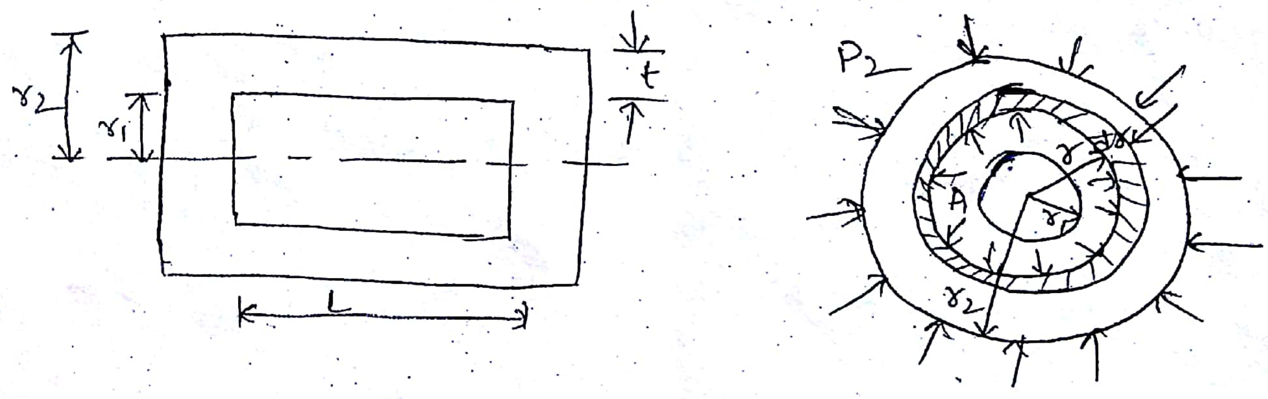
→ The variation in the radial as well as circumferential stresses across the thickness are obtained with the help of Lame's theory.

Lame's theory

Assumptions made in Lame's theory are :

1. The material is homogeneous & isotropic.
2. plane section perpendicular to the longitudinal axis of the cylinder remain plane after the application of internal pressure. (ε_L = const.)
3. The material is stressed within the elastic limit
4. All the fibres of the material are free to expand (or) contract independently without being constrained by the adjacent fibres.

consider a thick cylindrical shell of internal radius r_1 & external radius r_2 :



consider a elemental ring of radius r from centre and thickness dr.

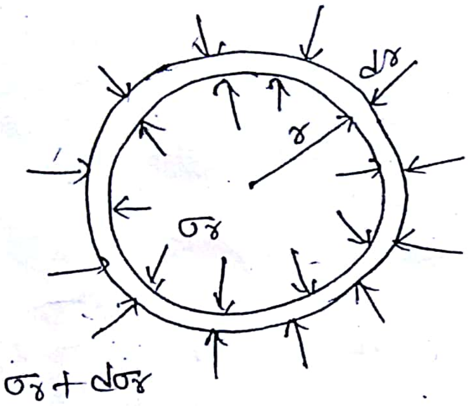
Let r_1 - internal radius of the cylinder
r_2 - External "
P_1 - internal pressure
P_2 - External pressure.

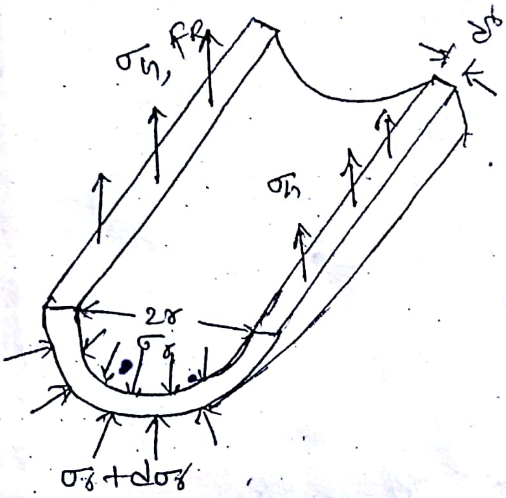
σ_r - internal radial stress (pressure) on the elemental ring.

$\sigma_r + d\sigma_r$ - External radial stress on the elemental ring

σ_h - circumferential stress.

Consider one half of the elemental ring.





In limiting case,

Bursting force = Resisting force

$$\text{Bursting force} = \sigma_r \times 2r \times L - (\sigma_r + d\sigma_r) \times 2(r + dr) \times L$$

$$= \sigma_r \times 2rL - (\sigma_r + d\sigma_r) \times (2Lr + 2L\,dr)$$

$$= 2r\cancel{L}\sigma_r - 2r\cancel{L}\sigma_r - 2L\,dr\,\sigma_r - 2rL\,d\sigma_r - 2L\,\cancelto{0}{dr}\,d\sigma_r$$

$$= -2L\,dr\,\sigma_r - 2rL\,d\sigma_r \quad [\because \text{smaller terms are negligible}]$$

$$= +2L\left[-\sigma_r\,dr - r\,d\sigma_r\right]$$

Resisting force = $F_R + F_R = 2F_R$

$= 2\sigma_h \times dr \times L$

∴ Bursting force = Resisting force.

$$+2L\left[-\sigma_r dr - r\,d\sigma_r\right] = 2L\,\sigma_h\,dr$$

$$\sigma_h = -\sigma_r \frac{\cancel{dr}}{\cancel{dr}} - r\frac{d\sigma_r}{dr}$$

$$\sigma_h = -\sigma_r - r\frac{d\sigma_r}{dr} \longrightarrow (1)$$

Now, let us obtain another relation b/w the radial stress & circumferential stress by using the condition that the longitudinal strain at any point in the section is same.

$$\text{longitudinal stress, } \sigma_L = \frac{P_1 \times \cancel{\pi} r_1^2}{\cancel{\pi}(r_2^2 - r_1^2)}$$

$$= \frac{P_1 r_1^2}{r_2^2 - r_1^2}$$

Hence at any point in the section of the elemental ring considered above, the following threes principal stresses exist.

1. radial (compressive) stress, σ_r (-ve)
2. circumferential (tensile) stress, σ_h +ve
3. Longitudinal (tensile) stress, σ_L. +ve

since the longitudinal strain (ε_L) is constant, we have

$$\varepsilon_L = \frac{\sigma_L}{E} - \mu\frac{\sigma_h}{E} + \mu\frac{\sigma_r}{E}$$

σ_L, E & μ are constant

$$\therefore \quad -\sigma_h + \sigma_r = \text{constant}$$

$$\sigma_h - \sigma_r = \text{constant}$$

let

$$\sigma_h - \sigma_r = 2a$$

$$\sigma_h = \sigma_r + 2a \longrightarrow (2)$$

(1) = (2)

$$-\sigma_r - r\frac{d\sigma_r}{dr} = \sigma_r + 2a$$

$$-r\frac{d\sigma_r}{dr} = 2\sigma_r + 2a$$

$$\frac{d\sigma_r}{dr} = \frac{2(\sigma_r + a)}{-r}$$

$$\frac{d\sigma_r}{\sigma_r + a} = -2\frac{dr}{r}$$

Apply integration on both sides

$$\log_e(\sigma_r + a) = -2\log_e(r) + \log_e(b)$$

$$= \log_e(b/r^2)$$

$$\therefore \quad \sigma_r + a = \frac{b}{r^2}$$

$$\boxed{\sigma_r = \frac{b}{r^2} - a} \longrightarrow (3)$$

sub (3) in (2), we get

$$\sigma_h = \frac{b}{r^2} - a + 2a$$

$$\boxed{\sigma_h = \frac{b}{r^2} + a} \longrightarrow (4)$$

(3) & (4) are Lame's equations.

The constants a & b can be evaluated from the known internal & external radial pressure and radius

<u>Boundary conditions</u>

<u>case-1</u>

If $r = r_1$, $\sigma_r = P_1$

$r = r_2$, $\sigma_r = 0$

sub in eqn (3)

$$P_1 = \frac{b}{r_1^2} - a$$

$$a = \frac{b}{r_1^2} - P_1 \longrightarrow (i)$$

$$0 = \frac{b}{r_2^2} - a$$

$$a = \frac{b}{r_2^2} \longrightarrow (ii)$$

(i) = (ii)

$$\frac{b}{r_1^2} - P_1 = \frac{b}{r_2^2}$$

$$P_1 = \frac{b}{r_1^2} - \frac{b}{r_2^2}$$

$$= b\left[\frac{r_2^2 - r_1^2}{r_1^2 r_2^2}\right]$$

$$b = \frac{P_1 r_1^2 r_2^2}{r_2^2 - r_1^2}$$

∴ put value of b in eqn (21)

$$a = \frac{\frac{P_1 r_1^2 \cancel{r_2^2}}{r_2^2 - r_1^2}}{\cancel{r_2^2}}$$

$$a = \frac{P_1 r_1^2}{r_2^2 - r_1^2}$$

Put a & b values in eqn (3) & (4), we get.

$$\sigma_r = \frac{\frac{P_1 r_1^2 r_2^2}{r_2^2 - r_1^2}}{r^2} - \frac{P_1 r_1^2}{r_2^2 - r_1^2}$$

$$\boxed{\sigma_r = \frac{P_1 r_1^2}{r_2^2 - r_1^2}\left(\frac{r_2^2}{r^2} - 1\right)}$$

$$\sigma_h = \frac{\frac{P_1 r_1^2 r_2^2}{r_2^2 - r_1^2}}{r^2} + \frac{P_1 r_1^2}{r_2^2 - r_1^2}$$

$$\boxed{\sigma_h = \frac{P_1 r_1^2}{r_2^2 - r_1^2}\left[\frac{r_2^2}{r^2} + 1\right]}$$

At $r = r_1$,

$$(\sigma_c)_{r_1} = P_1 \cdot \left(\frac{r_2^2 + r_1^2}{r_2^2 - r_1^2}\right)$$

At $r = r_2$

$$(\sigma_c)_{r_2} = P_1\left(\frac{2 r_1^2}{r_2^2 - r_1^2}\right)$$

<u>case-2</u>

$r = r_1$, $\sigma_r = 0$

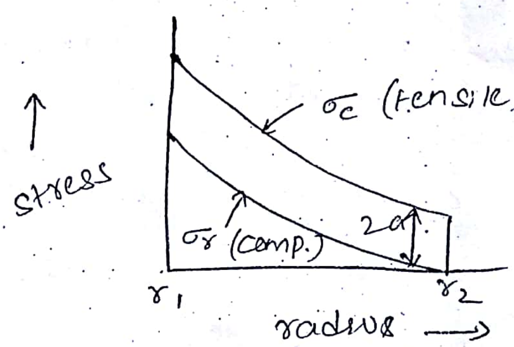
$r = r_2$, $\sigma_r = P_2$

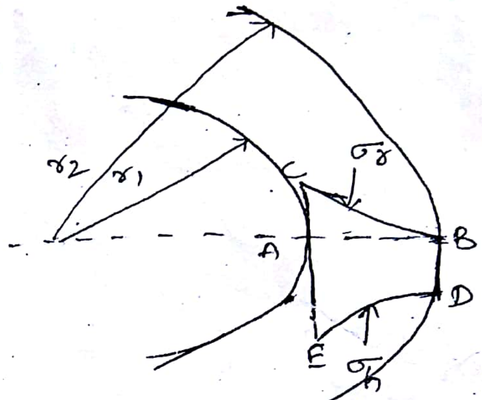
$a = ?$, $b = ?$

<u>case-3</u>

$r = r_1$, $\sigma_r = P_1$

Fig. shows the graph b/w stress vs radius. It is evident from the graph that the maximum values of both σ_r & σ_c occur at the inner surface.





radial stress & hoop stress variations

Longitudinal & shear stresses

$$\sigma_L = \frac{\text{Force acting on the end cover due to internal pressure}}{\text{Area of cross-section of the cylinder.}}$$

$$= \frac{P_1 \times \pi r_1^2}{\pi(r_2^2 - r_1^2)} = \frac{P_1 r_1^2}{r_2^2 - r_1^2}$$

$\sigma_h > \sigma_L > \sigma_r$.

(∵ No Torque)

Maximum shear stress

$$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2}$$

$$= \frac{\sigma_h - (-\sigma_r)}{2} = \frac{\sigma_h + \sigma_r}{2}$$

$$= \frac{\frac{b}{r^2} + a + \frac{b}{r^2} - a}{2}$$

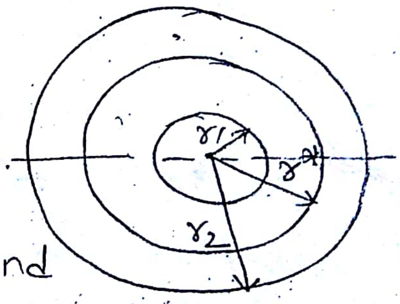
$$\boxed{\tau_{max} = \frac{b}{r^2}}$$

Compound cylinders :

In thick cylinders, the maximum hoop stress occurs at the inner ~~surface~~ circumference and it decreases towards the outer circumference. Hence the maximum pressure inside the shell is limited corresponding to the condition that the hoop stress at the inner circumference reaches the permissible value.

But, suppose the shell is made by shrinking the one cylinder over the other. Due to this, the inner cylinder will be put into initial compression whereas the outer cylinder will be put into initial tension. If now the compound cylinder is subjected to internal fluid pressure, both the inner & outer cylinders will be subjected to hoop tensile stress. The net effect of the initial stresses due to shrinking & those due to internal fluid pressure is to make the resultant stresses more or less uniform.

Fig. shows a compound thick cylinder made up of two cylinders.



Let r_1 - inner radius of compound cylinder

r_2 – outer radius of compound cylinder.

r^* – Radius at the junction of two cylinders.

p^* - Radial pressure at the junction of two cylinders.

Initial stresses

(a) For outer cylinder

$$\sigma_r = \frac{b}{r^2} - a \rightarrow ①, \quad \sigma_h = \frac{b}{r^2} + a \rightarrow ②$$

$0 = \frac{b}{(r_2^*)^2} - a \Rightarrow a = \frac{b}{r_2^2}$

At $r = r_2$, $\sigma_r = 0$.

$r = r^*$, $\sigma_r = p^*$

$$0 = \frac{b}{r_2^2} - a, \rightarrow (i) \quad p^* = \frac{b}{r^{*2}} - a \rightarrow (ii)$$

From equ (i) & (ii), constants a & b can be determined. These values are substituded in equ ②. and then hoop stresses in the outer cylinder due to shrinking can be obtained.

(b) For inner cylinder.

At $r = r_1$, $\sigma_r = 0$

$r = r^*$, $\sigma_r = p^*$

$$0 = \frac{b}{r_1^2} - a, \rightarrow (iii) \quad p^* = \frac{b}{r^{*2}} - a \rightarrow (iv)$$

~~substituting these values in equ ①, we get~~

From equ (iii) & (iv), constants a & b can be determined These values are substituted in equ ② & then hoop stresses are obtained.

(12)

Due to internal fluid pressure alone :

$$\sigma_r = \frac{B}{r^2} - A \quad , \quad \sigma_h = \frac{B}{r^2} + A .$$

At $r = r_2$, $\sigma_r = 0$

$r = r_1$, $\sigma_r = P_1$

$$0 = \frac{B}{r_2^2} - A \longrightarrow (v)$$

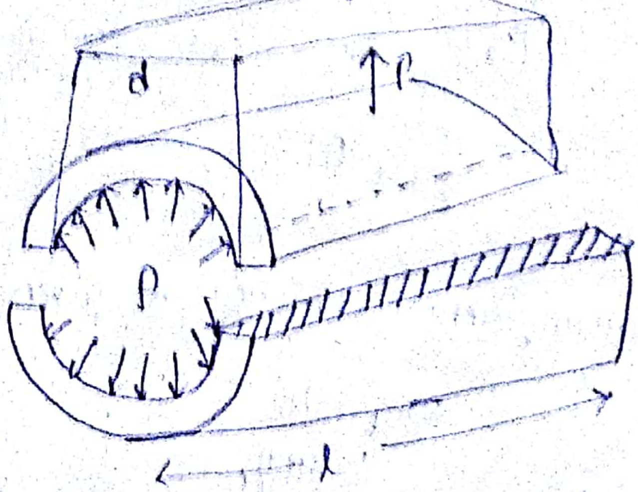
$$P_1 = \frac{B}{r_1^2} - A \longrightarrow (vi)$$

From (v) & (vi), constants A & B can be determined. These values are substituted in eqn ②, and then hoop stresses across the section can be obtained.

The resultant hoop stress will be the algebraic sum of the hoop stresses caused due to shrinking & those due to internal fluid pressure.

THIN CYLINDERS
UNIT-5

- If the thickness of the cylinder is less than $1/20^{th}$ of its diameter then they are called thin cylinders.
- If the thickness of the cylinder is more than $1/20^{th}$ of its diameter then they are thick cylinders.
- Circumferential stress (σ_c) or Hoop stress (σ_h):



External applied force = Internal resisting force

$$P \times A_P = \sigma_c \times A_R$$

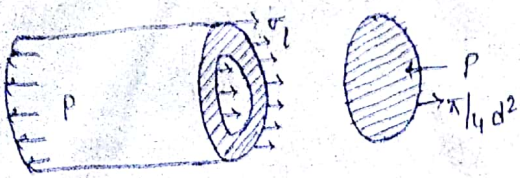
$$P \times l \times d = \sigma_c \times 2 \times (l \times t)$$

$$\sigma_c = \frac{Pd}{2t}$$

If longitudinal efficiency is given

$$\sigma_h = \frac{Pd}{2t\eta_l}$$

Longitudinal stress(σ_l)



External applied force = Internal resisting force

$$P \times A_p = \sigma \times A_l$$

$$P \times \frac{\pi}{4} d^2 = \sigma_l \times \pi d \times t$$

$$\sigma_l = \frac{Pd}{4t}$$

If circumference efficiency is given

$$\sigma_l = \frac{Pd}{4t\eta_c}$$

A cylindrical ~~wheel~~ Shell is 3m long having 1m inner diameter and 15mm thickness. Calculate the maximum intensity of shear stress & also the changes in the dimensions of the ~~wheel~~ Shell, if it is subject to an internal fluid pressure of 1.5 Mpa ($E = 2\times10^5 N/mm^2$, $\mu = 0.3$)

Given

$d_i = 1000$ mm

~~$d_o = 1015$ mm~~ $t = 15$mm

$l = 3000$ mm

$P = 1.5$ ~~xx~~ N/mm^2

Hoop stress $\sigma_c = \frac{Pd}{2t}$

$$= \frac{1.5 \times 1000}{2 \times 15}$$

$$= 50 \ N/mm^2$$

longitudnal stress $\sigma_l = \frac{Pd}{4t}$

$$= \frac{1.5 \times 1000}{4 \times 15}$$

$$= 25 N/mm^2$$

Hoop strain $\epsilon_c = \frac{\sigma_c}{E} - \frac{\mu \sigma_l}{E}$

$$= \frac{50}{E} - \frac{25\mu}{E}$$

$$= \frac{25}{E}(2-\mu) \longrightarrow (1)$$

Hoop strain $= \frac{\delta d}{d}$ ($\because$ In general) $\rightarrow (2)$

From (1) & (2)

We get,

$$\epsilon_c = \frac{\delta d}{d} = \frac{25}{E}(2-\mu) = \frac{25}{2\times10^5}(2-0.3) = 21.25\times10^{-5}$$

$$\delta d = 0.2125$$

longitudnal strain $\epsilon_l = \frac{\sigma_l}{E} - \frac{\mu\sigma_c}{E}$

$$= \frac{25}{E} - \frac{50\mu}{E}$$

$$= \frac{25}{E}(1-2\mu) \longrightarrow (3)$$

Longitudinal strain $= \frac{\delta l}{l}$ ($\because$ In general) $\rightarrow$ (4)

From (3) & (4)

we get

$$E_l = \frac{\delta l}{l} = \frac{25}{E}(1-2u) = \frac{25}{2\times10^5}(1-0.6) = 5\times10^{-5}$$

$\delta l = 0.15$

Volumetric strain $= \frac{\delta V}{V}$ ($\because$ In general)

$$V = \frac{\pi}{4}d^2 l = \frac{\pi}{4}(1000)^2 \cdot 3000 = 2356194490$$

$$\delta V = \frac{\pi}{4}[(2d)\cdot l\cdot \delta d + d^2 \delta l]$$

$$= \frac{\pi}{4}[2(1000)(3000)\cdot \delta d + (1000)^2 \delta l]$$

$$= \frac{\pi}{4}(6\times10^6 \cdot \delta d + 10^6 \delta l)$$

$$= \frac{\pi}{4}\left\{6\times10^6 \times \frac{25d}{E}(2-u) + 10^6 \times \frac{25l}{E}(1-2u)\right\}$$

$$= \frac{\pi}{4}\left[150\times10^9 \frac{(2-u)}{E} + 75\times10^9 \frac{(1-2u)}{E}\right]$$

~~$\frac{\pi}{4}\times10^9$~~ ~~$\frac{\pi}{4}$~~

~~$\delta V = 2E_d + E_l$~~

$$= \frac{\pi}{4}\left[\left(\cancel{150}^{75}\times10^9\right)\frac{(2-0.3)}{2\times10^5} + (75\times10^9)\frac{(1-2(0.3))}{2\times10^5}\right]$$

$$\frac{\pi}{4}[(75\times10^4\times1.7) + (75\times10^4\times0.2)]$$

$$= \frac{\pi}{4}(127.5\times10^4 + 15\times10^4)$$

$$= \frac{\pi}{4}(1425000)$$

$$= 1119192.383$$

Q. A thin cylindrical shell 2m long has 200mm diameter and thickness of metal 10mm it is filled completely with a fluid at atmospheric pressure. If an additional volume 25000 mm³ fluid is pumped in, find the pressure developed and a hoop stress developed and also changes in diameter and length ($E = 2\times10^5$ N/mm², $u = 0.3$).

Given

$l = 2m = 2000$ mm

$d = 200$ mm

$t = 10$ mm

$\delta V = 25000$ mm³

$E = 2\times10^5$ N/mm²

$u = 0.3$

$P = ?$

$\sigma_c = ?$

$\delta d = ?$

$\delta l = ?$

$$\sigma_c = \frac{Pd}{2t} = \frac{P\cdot 200}{2\cdot 10} = 10P$$

$$\sigma_c = 10P$$

$$\sigma_t = \frac{Pd}{4t} = \frac{P \times 200}{4 \times 10} = 5p$$

$$\sigma_t = 5p$$

$$e_c = \frac{Pd}{4tE}(2-\mu) \qquad \left[\because E_c = \frac{\sigma_c}{E} - \frac{\mu \sigma_t}{E}\right]$$

$$e_c = \frac{P \cdot 200}{4 \times 10 \cdot 2 \times 10^5}(2-0.3) \qquad \frac{Pd}{2tE} - \frac{Pd}{4tE}\mu$$

$$e_c = 4.25 \times 10^{-5} \times p \qquad \frac{Pd}{4tE}(2-\mu)$$

$$e_l = \frac{Pd}{4tE}(1-2\mu) \qquad \left[\because E_l = \frac{\sigma_l}{E} - \frac{\mu \sigma_c}{E}\right]$$

$$= \frac{P \cdot 200}{4 \times 10 \times 2 \times 10^5}(1-0.6) \qquad \frac{Pd}{4tE} - \mu\frac{Pd}{2tE}$$

$$e_l = 10^{-5} \times p \qquad \frac{Pd}{4tE}(1-2\mu)$$

Volumetric strain $= \frac{\delta V}{V}$

$$\frac{\delta V}{V} = 2e_c + e_l$$

$$V = \frac{\pi}{4} \times d^2 l = \frac{\pi}{4} \times 200^2 \times 2000$$

$$= 62831853.07$$

$$\frac{25000}{62831853.07} = (8.5 \times 10^{-5} \times p) + (10^{-5} \times p)$$

$$95 \times 10^{-5} \times p = 0.000397$$

$$p = \frac{0.000397}{9.5} \times 10^5$$

$$= 0.0000417 \times 10^5$$

$$p = 4.17 \text{ N/mm}^2$$

$$e_c = \frac{\delta d}{d} = 4.25 \times 10^{-5} \times p$$

$$\delta d = 4.25 \times 10^{-5} \times 4.17 \times 200$$

$$\delta d = 0.03 \text{ mm}$$

$$e_l = \frac{\delta l}{l} = 10^{-5} \times p$$

$$\delta l = 10^{-5} \times 4.17 \times 2000$$

$$\delta l = 0.08 \text{ mm}$$

$$\sigma_c = 10p$$

$$= 10 \times 4.17$$

$$= 41.7 \text{ N/mm}^2$$

Q. A shell 3.25 m long and 1 mm diameter is subjected to an internal pressure of 1.2 N/mm² if the maximum thickness of the shell is 1 mm find the circumferential and longitudinal sources find also the maximum shear stress and change in dimensions of the shell ($E = 200$ kN/mm², $\mu = 0.3$)

Given

$$l = 3.25 \text{ m} = 3250 \text{ mm}$$

$$d = 1 \text{ mm}$$

$P = 1.2\ N/mm^2$

$t = 10mm$

$\sigma_c = ?$

$\sigma_l = ?$

$$\sigma_c = \frac{Pd}{2t}$$

$$= \frac{1.2 \times 1}{2 \times 10}$$

$$= 0.06\ N/mm^2$$

$$\sigma_l = \frac{Pd}{4t}$$

$$= \frac{1.2 \times 1}{4 \times 10}$$

$$= 0.03\ N/mm^2$$

$$\cancel{e_c = \frac{Pd}{4tE}}\ \cancel{([illegible])}$$

$$e_c = \frac{\sigma_c}{E} - \frac{\mu \sigma_l}{E}$$

$$= \frac{0.06}{2 \times 10^5} - (0.3)\frac{(0.03)}{2 \times 10^5}$$

$$= (0.03 - 0.0045) \times 10^{-5}$$

$$= 0.0255 \times 10^{-5}$$

$$e_l = \frac{\sigma_l}{E} - \frac{\mu \sigma_c}{E}$$

$$= \frac{0.03}{2 \times 10^5} - (0.3)\frac{(0.06)}{2 \times 10^5}$$

$$= (0.015 - 0.009) \times 10^{-5}$$

$$= 0.006 \times 10^{-5}$$

$$e_c = \frac{\delta d}{d} = 0.0255 \times 10^{-5}$$

$$\delta d = 0.0255 \times 10^{-5} \times d$$

$$= 0.0255 \times 1 \times 10^{-5}$$

$$= 0.0255 \times 10^{-5}\ mm$$

$$e_l = \frac{\delta l}{l} = 0.006 \times 10^{-5}$$

$$\delta l = 0.006 \times 10^{-5} \times l$$

$$= 0.006 \times 10^{-5} \times 3250$$

$$= 19.5 \times 10^{-5}\ mm$$

$$\frac{\delta V}{V} = 2e_c + e_l$$

$$= (0.051 + 0.006) \times 10^{-5}$$

$$\frac{\delta V}{V} = 0.057 \times 10^{-5}$$

$$\delta V = 0.057 \times 10^{-5} \times V$$

$$V = \frac{\pi}{4} \times d^2 \times l = \frac{\pi}{4} \times 1 \times 3250$$

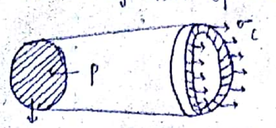
$$= 2552.54\ mm^3$$

$\delta V = 0.057 \times 10^{5} \times 2552.54$

$= 145.49 \times 10^{5}\ mm^3$

THIN SPHERES

• Stress and strain of thin spheres:



$\frac{\pi}{4} d^2$ (projected area)

Bursting force = resisting force

$p \times A_{projected} = \sigma_c \times A_{resisting}$

$p \times \frac{\pi}{4} d^2 = \sigma_c \times \pi d \times t$

$\sigma_1 = \frac{pd}{4t}$

$\sigma_2 = \frac{pd}{4t}$

$e_c = \frac{\sigma_1}{E} - \frac{\mu \sigma_2}{E}$

$= \frac{pd}{4tE} - \frac{pd}{4tE} \mu$

$= \frac{\sigma}{E}(1-\mu)$

$e_c = \frac{\delta d}{d} = \frac{pd}{4tE}(1-\mu)$

volumetric strain $e_v = \frac{\delta V}{V}$

$V = \frac{\pi}{6} d^3$

$\delta V = \frac{\pi}{6}(3d^2 \cdot \delta d)$

$e_v = \frac{\frac{\pi}{6}(3d^2 \cdot \delta d)}{\frac{\pi}{6} d^3}$

$= 3e_d$

$\frac{\delta V}{V} = e_v = \frac{3}{4} \frac{pd}{tE}(1-\mu)$

Formulae:

Thin cylinders

1. Hoop stress $\sigma_1 = \frac{pd}{2t}$

   $\sigma_1 = \frac{pd}{2t\eta_l}$

2. longitudinal stress $\sigma_2 = \frac{pd}{4t}$

   $\sigma_2 = \frac{pd}{4t\eta_c}$

3. Circumferential strain $e_1 = \frac{\delta d}{d} = \frac{pd}{4tE}(2-\mu)$

4. longitudinal strain $e_2 = \frac{\delta l}{l} = \frac{pd}{4tE}(1-2\mu)$

5. Volumetric strain $= 2e_2 + e_1 = \frac{\delta V}{V} = \frac{pd}{4tE}(5-4\mu)$

Spheres

1. Hoop stress = longitudinal stress

$$\sigma_1 = \sigma_2 = \sigma = \frac{Pd}{4t}$$

$$\sigma_1 = \sigma_2 = \sigma = \frac{Pd}{4t\eta}$$

2. Circumferential strain = $e_1 = e_2 = \frac{\delta d}{d} = \frac{Pd}{4tE}(1-\mu)$
longitudinal strain

3. Volumetric strain = $e_V = \frac{\delta V}{V} = \frac{3}{4}\frac{Pd}{tE}(1-\mu)$

$$V = \frac{\pi}{6}d^3$$

Q. At the atmospheric pressure a thin spherical shell has a diameter of 750mm and thickness 8 mm. Find the stress induced and the change in diameter, volume when the fluid pressure is increased to 2.5 N/mm² ($E = 2\times10^5$ N/mm², $\mu = 0.25$)

Given

$d = 750$ mm

$t = 8$ mm

$P = 2.5$ N/mm²

$E = 2\times10^5$ N/mm²

$\mu = 0.25$

$$\sigma = \frac{Pd}{4t} = \frac{2.5\times750}{4\times8}$$

$\sigma = 58.59$ N/mm²

$$e_s = \frac{\delta d}{d} = \frac{Pd}{4tE}(1-\mu)$$

$$\frac{2.5\times750}{4\times8\times2\times10^5}(1-0.25)$$

$$= \frac{1875}{64}(0.75)\times10^{-5}$$

$$\frac{\delta d}{d} = \underset{21.97}{[\text{illegible}]}\times10^{-5} \text{ mm} \qquad \delta d = 16477.5\times10^{-5}$$

$$e_V = \frac{\delta V}{V} = \frac{3}{4}\frac{Pd}{tE}(1-\mu)$$

$$V = \frac{\pi}{6}d^3$$

$$V = \frac{\pi}{6}(750)^3$$

$= \cancel{331339850.2}$ mm³
220893233.5

$$\delta V = \frac{3}{4}\times\frac{2.5\times750}{8\times2\times10^5}(1-0.25)\times V$$

$$\frac{3}{4}\times\frac{1875}{16}(0.75)\times \overset{220893233.5}{\cancel{331339850.2}}\times10^{-5}$$

$$= 87.89\times0.75\times\overset{220893233.5}{\cancel{331339850.2}}\times10^{-5}$$

145607.29
$= [\text{illegible}]$ mm³